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Exam I: MTH 111, Spring 2018

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$$\text{Points} = \frac{80}{80}$$

QUESTION 1. a) **(3 points)** Are the points $q_1 = (1, 2, -2)$, $q_2 = (3, 3, 1)$, and $q_3 = (5, 4, 4)$ co-linear? Show the work

$\overrightarrow{Q_1Q_2} = <2, 1, 3>$

$\overrightarrow{Q_1Q_3} = <4, 2, 6>$

$$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{vmatrix} = < \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix}, \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} > = <0,0,0>$$

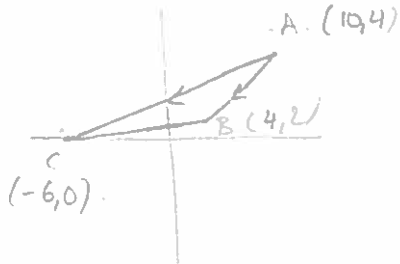
cross product is zero $\Rightarrow$ they are colinear

b) **(3 points)** Given $A = (10, 4)$, $B = (4, 2)$, and $C = (-6, 0)$ are the vertices of a triangle. Roughly, sketch the triangle ABC. Find the area of the triangle ABC.

$\overrightarrow{AB} = <-6, -2>$

$\overrightarrow{AC} = <-16, -4>$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -6 & -2 & 0 \\ -16 & -4 & 0 \end{vmatrix} = <0, 0, -8>$$



$$A_{\Delta ABC} = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{(-8)^2} = \boxed{4 \text{ units}^2}$$

c) **(3 points)** Find a vector F that is perpendicular to both vectors $V = <2, -1, 4>$ and $W = <0, 4, 2>$

$$\vec{F} = \vec{V} \times \vec{W} = \begin{vmatrix} i & j & k \\ 2 & -1 & 4 \\ 0 & 4 & 2 \end{vmatrix} = \boxed{<-18, -4, 8>}$$

d) **(2 points)** Let V, W as in (c). Find a vector F that is perpendicular to both V and W such that $|F| = 2$.(hint: Just think a little)

$$|F| = \sqrt{18^2 + 4^2 + 8^2} = 2\sqrt{101}$$

$$(2)\frac{1}{|F|} \cdot F = \frac{\cancel{2}}{\cancel{2}\sqrt{101}} \cdot F = \frac{1}{\sqrt{101}} \cdot F = \boxed{\frac{1}{\sqrt{101}} <-18, -4, 8>}$$

$$\boxed{F < \frac{-18}{\sqrt{101}}, \frac{-4}{\sqrt{101}}, \frac{8}{\sqrt{101}} >}$$

(check if $|F| = 2$:

$$|F| = \sqrt{\left(\frac{-18}{\sqrt{101}}\right)^2 + \left(\frac{-4}{\sqrt{101}}\right)^2 + \left(\frac{8}{\sqrt{101}}\right)^2} = 2$$)

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QUESTION 2. a) **(4 points)** Does the line $L_1 : x = 5t - 20, y = -t + 3, z = 3t - 27$ ($t \in R$) intersect the line $L_2 : x = -2w + 20, y = -4w - 5, z = 2w - 3$ ($w \in R$)? If yes find the intersection point Q.

$L_1: \begin{cases} x = 5t - 20 \\ y = -t + 3 \\ z = 3t - 27 \end{cases}$ $L_2: \begin{cases} x = -2w + 20 \\ y = -4w - 5 \\ z = 2w - 3 \end{cases}$

$5t - 20 = -2w + 20 \Rightarrow 5t + 2w = 40$

$-t + 3 = -4w - 5 \Rightarrow -t + 4w = -8$

$t = 8 \quad w = 0$

check for z:

$z = 3t - 27 = 3(8) - 27 = -3$

$z = 2w - 3 = 2(0) - 3 = -3$

} they are equal $\Rightarrow$ L_1 and L_2 intersect

The point of intersection

$x = 2w + 20 = 2(0) + 20 = 20$

$y = -4w - 5 = -4(0) - 5 = -5$

$z = 2w - 3 = 2(0) - 3 = -3$

point of intersection is $(20, -5, -3)$ ✓

b) **(2 points)** Are the lines in (a) perpendicular? Explain Yes.

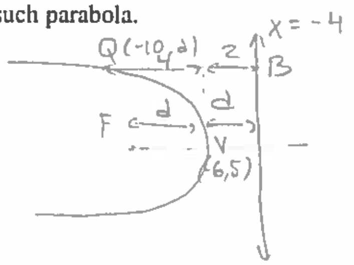
$D_1 = \langle 5, -1, 3 \rangle \quad D_2 = \langle -2, -4, 2 \rangle$

$D_1 \cdot D_2 = 5(-2) - 1(-4) + 3(2) = 0$ ✓

dot product $= 0 \Rightarrow$ They are perpendicular.

QUESTION 3. Given $x = -4$ is the directrix of of a parabola that has the point $(-6, 5)$ as its vertex point.

a) **(2 points)** Roughly, sketch such parabola.



$|d| = 2$

b) **(4 points)** Find the equation of the parabola

$4d(x - x_0) = (y - y_0)^2$

$-4(2)(x + 6) = (y - 5)^2$

$\boxed{-8(x + 6) = (y - 5)^2}$ ✓

c) **(2 points)** Find the focus of the parabola, say F.

$\boxed{F(-8, 5)}$ ✓

d) **(2 points)** Given $Q = (-10, b)$ is a point on the curve of the parabola. Find $|QF|$ (HINT: You should know how to do this QUICKLY!, you do not need the value of b)

$|QL| = |QB| = |QF| = \boxed{6}$ ✓

QUESTION 4. Given $y = x^2 - 6x - 1$ is an equation of a parabola.

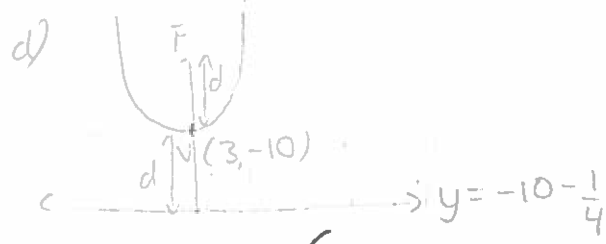
a)**(3 points)** Write the equation in the standard form.

$y = (x-3)^2 - 9 - 1$

$y = (x-3)^2 - 10$

$\boxed{(y+10) = (x-3)^2}$

$4d = 1 \Rightarrow d = 1/4$



b) **(2 points)** Find the equation of the directrix line.

$\boxed{y = -10 - \frac{1}{4} = -\frac{41}{4}}$

c)**(2 points)** Find the focus, say F

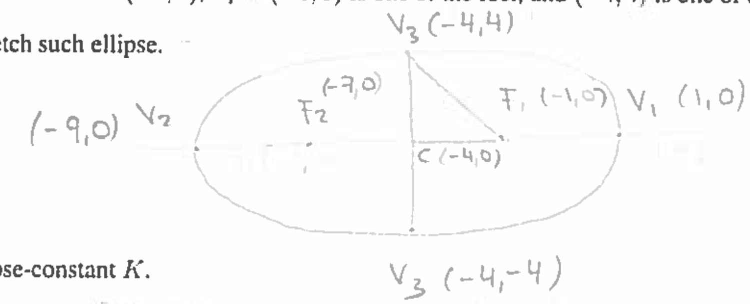
$\boxed{F(3, -10 + \frac{1}{4})} \to \boxed{F(3, -\frac{39}{4})}$

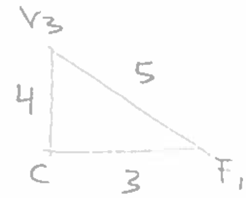
d)**(2 points)** Roughly, sketch the graph of such parabola.

(see picture)

QUESTION 5. An ellipse is centered at $(-4, 0)$, $F_1 = (-1, 0)$ is one of the foci, and $(-4, 4)$ is one of the vertices.

(i) **(2 points)** Roughly, sketch such ellipse.





(ii) **(3 points)** Find the ellipse-constant K.

$|V_3F_1| = \frac{K}{2} = 5 \Rightarrow \boxed{K = 10}$

(iii) **(2 points)** Find the second foci of the ellipse.

$\boxed{F_2(-7, 0)}$

(iv) **(3 points)** Find the remaining three vertices of the ellipse

$\boxed{V_3(-4, -4)}$

$\boxed{V_1(1, 0)}$

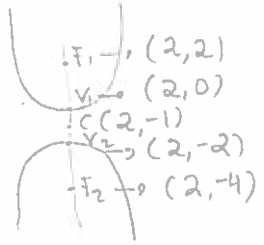
$\boxed{V_2(-9, 0)}$

(v) **(3 points)** Find the equation of the ellipse.

$$\boxed{\frac{(x+4)^2}{25} + \frac{y^2}{16} = 1}$$

QUESTION 6. Consider the hyperbola $(y+1)^2 - \frac{(x-2)^2}{8} = 1$.

a) **(2 points)** Draw the hyperbola, roughly



$|CF_1| = \sqrt{1+8} = 3$

b) **(2 points)** Find the hyperbola-constant K.

$\left(\frac{K}{2}\right)^2 = 1$

$\frac{K}{2} = 1 \Rightarrow \boxed{K = 2}$ ✓

c) **(3 points)** Find the two vertices of the hyperbola.

$\boxed{V_1\ (2,0)}$

$\boxed{V_2\ (2,-2)}$ ✓

d) **(3 points)** Find the foci of the hyperbola.

$\boxed{F_1\ (2,2)}$

$\boxed{F_2\ (2,-4)}$ ✓

QUESTION 7. Given two lines $L_1 : x = t+1, y = 2t+4, z = -5t+3$ and $L_2 : x = 2w+7, y = 4w+16, z = -10w-27$.

(i) **(3 points)** Find the symmetric equation of L_1.

$\boxed{x-1 = \frac{y-4}{2} = \frac{-z+3}{5}}$ ✓

(ii) **(3 points)** Is D_1 parallel to D_2? (note that D_1 is the directional vector of L_1 and D_2 is the directional vector of L_2) Show the work

$D_1 < 1, 2, -5 >$

$D_2 < 2, 4, -10 >$

$D_1 = c\,D_2$

$<1, 2, -5> = c <2, 4, -10>$

$c = \frac{1}{2}$ ✓

$D_1 = \frac{1}{2} D_2 \Rightarrow$ They are parallel

(iii) **(2 points)** Is L_1 parallel to L_2? Explain (show the work)

Take $t=0 \rightarrow (1,4,3)$

check if $(1,4,3) \in L_2$

$1 = 2w+7 \Rightarrow w = -3$

$4 = 4w+16 \Rightarrow w = -3$

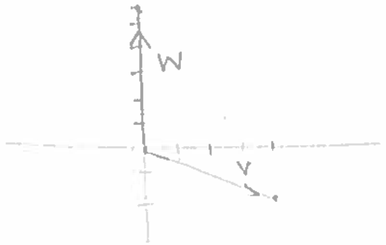
$3 = -10w-27 \Rightarrow w = -3$

$\Rightarrow$ it $\in$ to L_2.

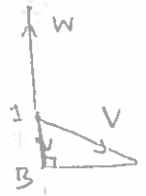
$\Rightarrow L_1$ and L_2 intersect and they are NOT parallel. They are collinear (same line/ on top of each other) ✓

QUESTION 8. Let (0, 0) be the initial point of the two vectors $V = < 4, -2 >$, and $w = < 0, 6 >$.

a) **(2 points)** Draw V and W in the xy-plane.

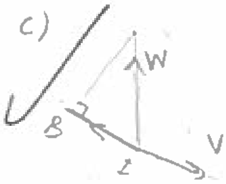


b)



$\text{proj}_W^V = \overrightarrow{1B}$

c)



$\text{proj}_V^W = \overrightarrow{1B}$

b) **(2 points)** Use the picture that you draw in (a) in order to draw $Proj_W^V$

c)**(2 points)** Use the picture that you draw in (a) in order to draw $Proj_v^w$

d) **(4 points)** Find $Proj_w^v$ and find its length.

$$\text{proj}_W^V = \frac{V \cdot W}{|W|^2} \cdot W = \frac{-12}{36} \cdot W = -\frac{1}{3} <0, 6> = \boxed{<0, -2>}$$

$$\boxed{|\text{proj}_W^V| = \sqrt{2^2} = 2}$$

e)**(3 points)** Find the angle between V and W

$$\cos\theta = \frac{V \cdot W}{|V||W|} = \frac{-12}{(6)(2\sqrt{5})} = -\frac{\sqrt{5}}{5}$$

$$\theta = \cos^{-1}\left(-\frac{\sqrt{5}}{5}\right) = \boxed{116.565^\circ}$$

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